Does there exist a graph with 5 vertices which have the following degrees: 2, 4, 4, 4, 4?

Ans: Yes.

1)Edges perspective: The upper bound edge number for a graph with n vertices is n \* (n-1) / 2 and lower bound edge number is n – 1 (tree). In this case, the number of edges should great than 4 and less than 10. Sum of degree = 18. Number of edges = 9. The edges is greater than 4 \* 3 / 2 = 6, the graph is connected/ exist. No problem in this aspect.

2) No problem in handshaking Lemma.

3) Havel- Hakimi Theorem (which is applicable to simple graphs). Because there is no -1 appears in the whole process. No problem in this aspect.

Order: 4,4,4,4,2

Delete: 3,3,3,1

Delete: 2,2,0

Delete: 1,0

Delete: 0

A picture containing bird

Description automatically generated

Hence, the graph exists.